ECON 4151
Lab Session 3: Simultaneous Equations Models and Logit Model

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Outline

Why regressors and error term are correlated?

1. Measurement errors in the regressor(s)
2. Omitted variable bias
3. Simultaneous equation bias
4. Dynamic regression model with serial correlation in the error term

Simultaneity bias in OLS

- Two-equation structural model:

$$y_1 = \alpha_1 y_2 + u_1$$
$$y_2 = \alpha_2 y_1 + \beta_2 z_2 + u_2$$

where z_2 is exogenous and u_1 and u_2 are uncorrelated.

- Focus on estimating the first equation.
- We can show that by plugging the RHS of the first eq'n in for y_1 in the second eq'n,

$$\text{Cov}\left(y_2, u_1\right) = \frac{\alpha_2}{1 - \alpha_2\alpha_1} \text{Var}\left(u_1\right)$$

- Just like OVB, the asymptotic bias (or inconsistency) in the OLS estimator of α_1 has the same sign as $\alpha_2 / \left(1 - \alpha_2\alpha_1\right)$.
- Why? See the next slide.

Simultaneity bias in OLS

- Let α_1 is the true parameter and a_1 is a naive OLS estimate w/o considering the SEM structure.
- Just like we did in OVB,

$$
\begin{aligned}
a_1 &= \frac{\text{Cov}(y_2, y_1)}{\text{Var}(y_2)} \\
&= \frac{\text{Cov}(y_2, \alpha_1 y_2 + u_1)}{\text{Var}(y_2)} \\
&= \alpha_1 + \frac{\text{Cov}(y_2, u_1)}{\text{Var}(y_2)} \\
&= \alpha_1 + \frac{\alpha_2}{1 - \alpha_2 \alpha_1} \underbrace{\frac{\text{Var}(u_1)}{\text{Var}(y_2)}}_{>0}
\end{aligned}
$$

Simultaneity bias in OLS

- Simultaneity $\implies$ explanatory variables are endogenous $\implies$ OLS estimators are biased and inconsistent.
- This arises when one or more of the explanatory variables is jointly determined with the dependent variable, typically through an equilibrium mechanism.
- IV can fix it!

Identifying and estimating a structural equation

- A two-equation SEM in eq'm form ($q_s = q_d = q$ imposed):

$$q = \alpha_1 p + \beta_1 z_1 + u_1 \quad : \text{ supply equation}$$
$$q = \alpha_2 p + u_2 \quad : \text{ demand equation}$$

  where z_1 is exogenous and u_1 and u_2 are uncorrelated.
- z_1 is an IV for the price in the demand equation.
  - Why? z_1 is an exogenous shock to price in the demand equation (it is a supply shifter).
- The demand equation is identified, but the supply equation is not.

Stata excercise

- Wooldridge Ch. 16, C3: Inflation and openness

$$
\begin{aligned}
inf &= \beta_{10} + \alpha_1 open + \beta_{11}\log(pcinc) + u_1 \\
open &= \beta_{20} + \alpha_2 inf + \beta_{21}\log(pcinc) + \beta_{22}\log(land) + u_2
\end{aligned}
$$

- Wooldridge Ch. 16, C6: Demand function for airline seats on routes in the U.S.

$$\log(passen) = \beta_{10} + \alpha_1 \log(fare) + \beta_{11}\log(dist) + \beta_{12}[\log(dist)]^2 + u_1$$